On the origin of the different Mayan Calendars

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The Maya were known for their astronomical proficiency. This is demonstrated in the Mayan codices where ritual practices were related to astronomical events/predictions. Whereas Mayan mathematics were based on a vigesimal system, they used a different base when dealing with long periods of time, the Long Count Calendar (LCC), composed of different Long Count Periods: the Tun of 360 days, the Katun of 7200 days and the Baktun of 144000 days. There were two other calendars used in addition to the LCC: a civil year Haab' of 365 days and a religious year Tzolk'in of 260 days. Another cycle was also in use: a 3276-day cycle (combination of the 819-day Kawil cycle and the 4 directions-colors). Based on the hypothesis that Mayan astronomers had a good knowledge of naked-eyed astronomy, we propose here an explanation of the origin of the LCC, the Haab', the Tzolk'in and the 3276-day cycle. This study shed more light on the connection between astronomy, arithmetics and religion in Maya civilization.

I. INTRODUCTION

Mayan astronomy and religion have always been intertwined. This can be seen in the various codices where Mayan astronomers-priests were calculating the commensuration of astronomical and ritual cycles to determine the date of religious and civil ceremonies in a complex set of calendars and cycles. To date these ceremonies over long period of time and pinpoint historical events, the Maya used the Long Count Calendar (LCC), counting the number of day elapsed since the beginning time, the mythical date of creation. The LCC is organized as follow. The smallest unit of time is the day (Kin); 20 Kin form a Winal, 18 Winal form a Tun (360 Kin), 20 Tun form a Katun (7200 Kin), and 20 Katun form a Baktun (144000 Kin). The LCC represents a date D as a set of coefficients $(C_i, ..., C_3, C_2, C_1, C_0)$ such that: $D = C_0 + C_1 \times 20 + \sum_{i=2}^{n} C_i \times 18 \times 20^{i-1}$ with $C_0 = \text{mod}(D{,}20)$, $C_1 = \text{int}(\text{mod}(D{,}360)/20)$ and $C_i = \text{int}(\text{mod}(D, 18 \times 20^i)/(18 \times 20^{i-1})$ for $i > 1$. The day count usually restarts when C_4 reaches 13, such as the date is given as a set of 5 coefficients: $D \equiv \text{mod}(D, 13 \times 144000) = C_4.C_3.C_2.C_1.C_0$. Along with the LCC, the Maya used two other independent calendars: a religious year (Tzolk'in), a civil year (Haab'). One Tzolk'in of 260 days comprised 13 months (numerated from 1 to 13) containing 20 named days (Imix, Ik, Akbal, Kan, Chicchan, Cimi, Manik, Lamat, Muluc, Oc, Chuen, Eb, Ben, Ix, Men, Cib, Caban, Etznab, Cauac, and Ahau). This forms a list of 260 ordered Tzolk'in dates from 1 Imix, 2 Ik, ... to 13 Ahau.[1] One Haab' of 365 days comprised 18 named months (Pop, Uo, Zip, Zotz, Tzec, Xul, Yaxkin, Mol, Chen, Yax, Zac, Ceh, Mac, Kankin, Muan, Pax, Kayab, and Cumku) with 20 days each (Winal) plus 1 extra month (Uayeb) with 5 nameless days. This forms a list of 365 ordered Haab' dates from 0 Pop, 1 Pop, ... to 4 Uayeb.[2] The Tzolk'in and the Haab' coincide every 73 Tzolk'in or 52 Haab' or a Calendar Round (CR). A CR corresponds to the least common multiple (LCM) of 260 and 365: $73 \times 260 = 52 \times 365 = 18980$ days. In the Calendar Round, a date is represented by αXβY with the religious month $1 \leq \alpha \leq 13$, X one of the 20 religious days, the civil day $0 \leq \beta \leq 19$, and Y one of the 18 civil months, $0 \leq \beta \leq 4$ for the Uayeb. According to the Goodman-Martinez-Thompson (GMT) correlation to the Gregorian calendar, which is based on historical facts, the Mayan Calendar began on 11 August 3114 BC or 0(13).0.0.0.0 4 Ahau 8 Cumku and ended on 21 December 2012 or 0(13).0.0.0.0 4 Ahau 3 Kankin. This corresponds to a 13 Baktun cycle (1872000 days) or a period of approximately 5125 years. Another ritual cycle of 3276 days was used. It is formed by the combination of the Kawil, a 819-day cycle, and a 4-day direction-color cycle.[3] The direction-color decreases/changes for each Kawil as $\text{mod}(n \times 819 + 3{,}4) = 3, 2, 1$ and 0 (East-Red, South-Yellow, West-Black and North-White) for $n \in \mathbb{N}$ i.e. the 4-Kawil cycle starts at {3, East-Red}.

The origin of the Long Count Periods (LCPs) is unknown. A common assumption is the desire of the calendar keeper to maintain the Tun in close agreement with the tropical/solar year of approximately 365.24 days.[4] There is no consensus concerning the origin of the Tzolk'in, which has been associated with various astronomical cycles. Three Tzolk'in correspond to Mars synodic period, 16 Tzolk'in equal 11 of Saturn synodic periods (+2 days), and 23 Tzolk'in are equivalent to 15 Jupiter synodic periods (-5 days).[5] It has been tentatively connected to the eclipse half-year (173.31 days) because 2 Tzolk'in are very close to 3 eclipse half-years.[6] Finally, it has been noted that the Tzolk'in approximates the length of time Venus is visible as a morning or evening star.[7] However, these interpretations fail to link the Tzolk'in to the LCPs. The Kawil cycle has been attributed to the observation of Jupiter and Saturn[8,9] because 19 (6) Kawil correspond to 39 (13) Jupiter (Saturn) synodic period. Why the Calendar Round starts on a 4 Ahau 8 Cumku is still an opened question as is the starting count of the 4-Kawil cycle. Four numbers of possible astronomical significance have been discovered on the walls of a residential structure in Xultun, Guatemala and have been dated from the early 9$^{\text{th}}$ century CE. The Xultun numbers are given in Table I. They are such that $\mathcal{X}_1 = 365 \times 3276$ and

$\mathcal{X}_3 = \mathcal{X}_2 + 2\mathcal{X}_0$.

$\mathcal{X}_i$	LCC	D [day]	$\mathcal{X}_i/56940$
$\mathcal{X}_0$	2.7.9.0.0	341640	6
$\mathcal{X}_1$	8.6.1.9.0	1195740	21
$\mathcal{X}_2$	12.5.3.3.0	1765140	31
$\mathcal{X}_3$	17.0.1.3.0	2448420	43

TABLE I. Xultun numbers $\mathcal{X}_i$.[10] 56940 = LCM(365,780) is their largest common divisor.

Based on the assumption that Mayan astronomers observed the periodic movements of the five Planets visible to the naked eye, this paper explains a possible origin of the Long Count Periods, the religious year Tzolk'in and the relationship between the Kawil cycle and the 4 directions-colors. This hypothesis provides also a possible explanation of the date origin of the Calendar Round and the 4-Kawil cycle. The paper is organized as follow. In section II, we derive an important number based on astronomical observations and arithmetical considerations that seems to explain some aspects of Mayan religion and calendars. In section III, we study the various calendar cycles, some of them identified on Mayan Codices and monuments, and examine their relationship with the Planet canonic cycles. The last section is left for conclusion.

II. MAYAN ASTRONOMY AND RELIGION

Planet	P [day]	Prime factorization
Mercury	116	$2^2 \times 29$
Venus	584	$2^3 \times 73$
Earth	365	5×73
Mars	780	$2^2 \times 3 \times 5 \times 13$
Jupiter	399	$3 \times 7 \times 19$
Saturn	378	$2 \times 3^3 \times 7$
Lunar	177	3×59
senesters	178	2×89
Pentalunex	148	$2^2 \times 37$

TABLE II. Planet canonic cycles[11,12] and their prime factorizations.

The Maya were known for their astronomical skills as exemplified by the Dresden Codex, a bark-paper book of the 11$^{\mathrm{th}}$ or 12$^{\mathrm{th}}$ century CE. On page 24 of this Codex is written the so-called Long Round number noted 9.9.16.0.0 in the LCC or 1366560 days, a whole multiple of the Tzolk'in, the Haab', the Calendar Round, the Tun, Venus and Mars synodic periods: $\mathcal{LR}$ = 1366560 = 4 × $\mathcal{X}_0$ = 5256 × 260 = 3744 × 365 = 72 × 18980 = 3796 × 360 = 2340 × 584 = 1752 × 780. Only the moon, Mercury, Venus, Earth (solar year), Mars, Jupiter, and Saturn are visible to the naked eye. Their respective canonic synodic periods are given in Table II. Evidences have been found in different Mayan Codices that Mayan astronomers observed the periodic movements in the night sky of Mercury, Venus, and Mars, but it is unclear whether they tracked the movements of Jupiter and Saturn.[13] The periods relevant for the prediction of solar/lunar eclipses are the pentalunex of 148 days (5 Moon synodic periods of 29.53 days) and the lunar semesters of 177 or 178 days (6 Moon synodic periods), which are the time intervals between subsequent eclipse warning stations present in the Eclipse Table on pages 51 to 58 of the Dresden Codex.[12] The LCM of these numbers is $\mathcal{N}$ = 20757814426440 days (including the Planet synodic periods and the two lunar semesters) or $\mathcal{N}^\dagger$ = 768039133778280 days (also including the pentalunex). Table III gives the divisibility of $\mathcal{N}$ by a polynomial expression of the type $P_i = 13 \times 73 \times (\sum_n 18 \times 20^n)$.

Name	i	C_i [day]	$\mathcal{N}/P_i$	D_i
-	0	18	$\mathcal{N}/13/73/\sum_0^6 C_i$	18
Tun	1	360	$\mathcal{N}/13/73/\sum_0^5 C_i$	360
Katun	2	7200	$\mathcal{N}/13/73/\sum_0^4 C_i$	7215
Baktun	3	144000	$\mathcal{N}/13/73/\sum_0^3 C_i$	144304
Pictun	4	2880000	$\mathcal{N}/13/73/\sum_0^2 C_i$	2886428
Calabtun	5	57600000	$\mathcal{N}/13/73/\sum_0^1 C_i$	57866020
Kinchiltun	6	1152000000	$\mathcal{N}/13/73/C_0$	1215186420

TABLE III. Divisibility of $\mathcal{N}$ = 20757814426440 days by a polynomial expression of the type $P_i = 13 \times 73 \times (\sum_{n=0}^{6-i} 18 \times 20^n)$. $D_i = \mathrm{int}(\mathcal{N}/P_i)$.

$\mathcal{N}$ is such that:

$$\mathrm{int}(\mathcal{N}/13/73/144000) = 338 + 360 + 7200 + 144000. \quad (1)$$

$\mathcal{N}^\dagger$ gives the same results but divided by 13, 37, 73 and 144000. That defines the Tun, Katun, and Baktun as a polynomial expression of int($\mathcal{N}$/13/73) of the form $18 \times 20^3 \times (C_0 + \sum_{n=1}^3 18 \times 20^n)$ with $C_0 = 338$. The LCM(338,360) = 234 × 260 = 60840. 338 and 365 are relatively prime numbers: the LCM(338,365) = 338 × 365 = 123370. 234 and 365 are also relatively prime numbers: the LCM(234,365) = 234 × 365 = 85410. On the other hand, the Tzolk'in and the Haab' are commensurate: the LCM(260,365) is 73 × 260 = 52 × 365 = 18980 days or a Calendar Round. That may define the Tzolk'in. $\mathcal{N}$ is such that the mod($\mathcal{N}$,$\mathcal{LR}$) = 341640 days or 2.7.9.0.0 in the LCC. This is the Xultun number $\mathcal{X}_0$ (Table I). It is to be noted that $\mathcal{X}_0$ is a whole multiple of the Tzolk'in, Haab', Tun, Venus and Mars synodic periods: 341640 = 1314 × 260 = 936 × 365 = 949 × 360 = 585 × 584 = 438 × 780. $\mathcal{X}_0$ is also the largest common divisor of $\mathcal{LR}$ and $\mathcal{N}$.

Taking into account the ordered Tzolk'in and Haab' date,[1,2] a date D in the CR can be written as $D \equiv$ {mod(D + 160,260);mod(D + 349,365)} from the beginning of the 13 Baktun era 4 Ahau 8 Cumku {160;349}. We remark that mod($\mathcal{N}$/13/73,260) = 160 = 4 mod 13 = 0 mod 20 or the Tzolk'in date 4 Ahau, associating a number to the day in the Tzolk'in dates list.[1] We remark also that mod($\mathcal{N}$/13/73,73) = 49 or the Haab' date 8 Zip, associating a number to the day in the Haab' dates list.[2] The date 4 Ahau 8 Zip {160;49} may define the date origin of the CR. If we consider the Mayan conception of time as cyclical, this day 0 (18980) corresponds to the completion of a CR such that: mod(18980,4680) = 260 where 4680 = LCM(260,360) = 13 × 360 is the 13 Tun Wheel. The next date such that mod(D,4680) = 0 is the date 4 Ahau 8 Cumku {160;349}: mod(4680 + 160,260) = 160 and mod(4680 + 49,365) = 349. The Calendar Round began on a 4 Ahau 8 Zip, 4680 days earlier than the Long Count Calendar beginning date 0(13).0.0.0.0 4 Ahau 8 Cumku.

III. MAYAN CALENDAR AND RITUAL CYCLES

We first examine the cycles that can be constructed from the combination of the Tzolk'in (260 = 13 × 20), the Haab' (365 = 5 × 73) and the LCPs (D = 18 × 20^n). Because 13 is not present in the prime factorizations of the LCPs, we obtain LCM(260,D) = 13 ×D. There are, for example, the 13 Tun Wheel of 4680 days (18 Tzolk'in) and a 13 Katun cycle of 93600 days (360 Tzolk'in) reminiscent of the Katun Wheel. The 13 Baktun cycle comes from the coincidence between the Tzolk'in and the Baktun, corresponding to 1872000 days (400 × 13 Tun Wheel) or approximately 5125 years. Because 73 is not present in the prime factorizations of the LCPs, we obtain LCM(365,D) = 73 ×D. This gives rise to 73 LCP cycles. Moreover, the LCM(260,365,D) = 13 × 73 × D. This is the case of the Xultun number $\mathcal{X}_0$ = 341640 = LCM(260,365,360) which makes coincide the Tzolk'in, the Haab' and the Tun.

Planet	P [day]	N = LCM(P,260)	N/260	N/P
Mercury	116	7540	29	65
Venus	584	37960	146	65
Earth	365	18980	73	52
Mars	780	780	3	1
Jupiter	399	103740	399	260
Saturn	378	49140	189	130
Lunar	177	46020	177	260
semesters	178	23140	89	130
Pentalunex	148	9620	37	65

TABLE IV. Coincidence of the Planet canonic cycles[11,12] and the Tzolk'in.

Various religious cycles can be constructed by taking the LCM of 260 and the Planet synodic periods (Table IV). The most important one in Mayan religion is the Calendar Round, which is the LCM(260,365) = 73 × 260 = 52 × 365 = 18980 days. There is a coincidence between Venus synodic period and the Tzolk'in: the LCM(260,584) = 65 × 584 = 146 × 260 = 104 × 365 = 37960 days (2 Calendar Rounds), the length of the Venus Table on pages 24-29 of the Dresden Codex. There is also a coincidence between Mars synodic period and the Haab': the LCM(780,365) = 73 × 780 = 219 × 260 = 156 × 365 = 56940 days (3 Calendar Rounds): this is the largest common divisor of the Xultun numbers (Table I). We identify here two other cycles based on the coincidence of the Tzolk'in and the 234- and 338-day periods: the LCM(260,234) = 9 × 260 = 2340 days and the LCM(260,338) = 13 × 260 = 3380 days.

LCP	D [day]	L [day]	M [day]	N [day]	L/260	L/234
Winal	20	3380	260	2340	13	14.44444
Tun	360	60840	4680	4680	234	260
Katun	7200	1216800	93600	93600	4680	5200
Baktun	144000	24336000	1872000	1872000	93600	104000

TABLE V. Coincidence of the 234-, 260- and 338-day periods with the LCPs. L = LCM(D,338), M = LCM(D,260) and N = LCM(D,234).

Table V gives the coincidence of the 338-day period with the LCPs, as well as the Tzolk'in and the 234-day period. As described earlier, the Tun and 338-day period coincide every 60840 days, or 234 Tzolk'in. The 338-day period also coincides with the Winal every 3380 days, corresponding to 13 Tzolk'in, a cycle that does not coincide with the 234-day period. Rather, the 234-day period coincides with the Winal every 2340 days or 9 Tzolk'in = 9 × 260 = LCM(9,13,20). This 2340-day cycle is present in the Dresden Codex on pages D30c-D33c and has been attributed to a Venus-Mercury almanac because 2340 = 20 × 117 = 5 × 585 is an integer multiple of Mercury and Venus mean synodic periods (+1 day).[14] Another explanation may be of divination origin because 117 = 9 × 13. In Mesoamerican mythology, there are a set of 9 Gods called the Lords of the Night[15–18] and a set of 13 Gods called the Lords of the Day.[18] Each day is linked with 1 of the 13 Lords of the Day and 1 of the 9 Lords of the Night in a repeating 117-day cycle. This cycle coincides with the Tzolk'in every 2340 days. On Table V figures a date D = LCM(338,7200) = 1216800 days or 8.9.0.0.0 in the LCC or 5 February 219 CE according to the GMT correlation. This date may have marked the beginning of the reign of Yax Moch Xoc, the founder of the Tikal dynasty, in 219 CE.[19]

We then study the cycle created by the combination of the 819-day Kawil cycle and the 4-day direction-color cycle, coinciding every 3276 days. The Haab' and the Tzolk'in coincide with the 3276-day cycle every LCM(365,3276) = 365 × 3276 = 260 × 4599 = 1195740 days or 8.6.1.9.0 in the LCC. This is the Xultun number $\mathcal{X}_1$ (Table I). $\mathcal{X}_1$ = LCM(13,9,4,819,260,365) is the time distance between two days of the same Lord of the Day, Lord of the Night, direction-color, Kawil, Tzolk'in and Haab' date. $\mathcal{X}_0$ and $\mathcal{X}_1$ are such that 341640 × 819 = 1195740 × 360 = 2391480 or $\mathcal{Y}$ = 16.12.3.0.0 in the LCC. $\mathcal{Y}$ represents the same as $\mathcal{X}_1$ at the end of a Tun (0 Winal, 0 Kin). The Kawil coincides with the Tun every 32760 days = LCM(360,819) = 7 × LCM(260,360) = 10

Planet	P [day]	$N = \mathrm{LCM}(P,L)$	$N/260$	$N/2340$	N/L
Mercury	116	950040	3654	406	29
Venus	584	2391480	9198	1022	73
Earth	365	2391480	9198	1022	73
Mars	780	32760	126	14	1
Jupiter	399	622440	2394	266	19
Saturn	378	98280	378	42	3
Lunar	177	1932840	7434	826	59
semesters	178	2915640	11214	1246	89
Pentalunex	148	1212120	4662	518	37

TABLE VI. Coincidence of the 32760-day cycle with the Planet canonic cycles.[11,12] $N = \mathrm{LCM}(P,360,819)$ and $L = \mathrm{LCM}(360,819) = 32760$ days.

$\times$ 3276. Table VI shows the coincidence of the 32760-day cycle with the Planet canonic cycles, giving rise to whole multiples of the Tzolk'in and the 2340-day cycle. We remark that $\mathrm{mod}(N/32760,4) = 3$ which may explain the start of the 4-Kawil cycle. As can be seen in its prime factorization ($3276 = 2^2 \times 3^2 \times 7 \times 13$), the 4-Kawil cycle commensurates with all the Planet cycles except the Haab' (365 and 3276 are relatively prime numbers). We have for example the LCM(148,3276) = 37 $\times$ 3276 = 121212 days.

FIG. 1. Pyramid of Kukulkan during an equinox. The pyramid is situated at Chichen Itza, Yucatan, Mexico.

Finally, we discuss an important religious site in Mesoamerica, the pyramid of Kukulkan built sometime between the 9$^{\mathrm{th}}$ and the 12$^{\mathrm{th}}$ century CE at Chichen Itza (Figure 1) where various numbers in the architecture seem to have been related to calendrical considerations. The pyramid shape may be linked to the Long Count Calendar and the Planet canonic cycles (Table III) which draws a pyramid-like structure. It is constituted of 9 platforms with 4 stairways of 91 steps each leading to the platform temple corresponding to the 3276-day cycle: 3276 = LCM(4,9,91) = 4 $\times$ 819, the coincidence of the 4 directions-colors with the Kawil. The dimensions of the pyramid may be of significance: the width of its base is 55.30 m (37 zapal), its height up to the top of the platform temple is 30 m (20 zapal) and the width of the top platform is 19.52 m (13 zapal), taking into account the Mayan zapal length measurement such that 1 zapal $\approx$ 1.5 m.[20] The width of the base and 3276 are such that 37 $\times$ 3276 = LCM(148,3276) = 121212. The pyramid height and the width of the top platform represents the Tzolk'in. The stairways divide the 9 platforms of each side of the pyramid into 18 segments which, combined with the pyramid height, represents the 18 Winal of a Tun. The Haab' is represented by the platform temple making the 365$^{\mathrm{th}}$ step with the 4 $\times$ 91 = 364 steps of the 4 stairways. Each side of the pyramid contains 52 panels corresponding to the Calendar Round: 52 $\times$ 365 = 73 $\times$ 260 = 18980. During an equinox, the Sun casts a shadow (7 triangles of light and shadow) on the northern stairway representing a serpent snaking down the pyramid (Figure 1). A 1820-day cycle or 7 Tzolk'in = 5 $\times$ 364 is present on pages 31-32a of the Dresden Codex. This may also correspond to 32760 = LCM(360,819) = 7 $\times$ LCM(260,360).

IV. CONCLUSION

The study presented here describe a possible explanation of the origin of the Long Count Calendar (LCC) and its connection with the Tzolk'in and Haab' calendars. Based on arithmetical calculations on relevant astronomical periods obtained from naked-eyed observation of the Solar System, the LCC gives rise to three different periods of 234 days, 260 days (Tzolk'in) and 338 days. These astronomical calculations explain the various cycles used in Mayan religion and calendars. The 13 Baktun cycle originates from the coincidence between the Tzolk'in and the Baktun. The coincidence of the 234-day period and the Tzolk'in, a 2340-day cycle identified in the Dresden Codex, was certainly used for divination purposes. The coincidence of the 338-day period and the Tzolk'in gives rise to a 13-Tzolk'in cycle that has not yet been identified in the Codices. The 3276-day cycle, the coincidence of the 819-day Kawil and the 4 directions-colors, commensurates with all the Planet canonic cycles. These results indicate that Mayan astronomers calculated the synodic periods of all the Planets visible to the naked eye, as well as the basic cycles for solar/lunar eclipse prediction, the pentalunex and the lunar semesters.

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[1] A. F. Aveni, in Skywatchers: A Revised and Updated Version of Skywatchers of Ancient Mexico (University of Texas Press, 2001), p. 143.
[2] A. F. Aveni, in Skywatchers: A Revised and Updated Ver-

sion of Skywatchers of Ancient Mexico (University of Texas Press, 2001), p. 147.
[3] H. Berlin and D. H. Kelley, The 819-day Count and direction-color Symbolism among the Classic Maya. Middle American Research Institute Publication 26 (1961).
[4] A. F. Aveni, in Skywatchers: A Revised and Updated Version of Skywatchers of Ancient Mexico (University of Texas Press, 2001), p. 136.
[5] J. S. Justeson, The ancient Maya Ethnoastronomy: An Overview of Hieroglyphic Sources, in World Archaeoastromy: Selected Papers from the Second Oxford International Conference on Archaeoastronomy, edited by A. F. Aveni, (Cambridge University Press, 1989), p. 82.
[6] A. F. Aveni, in Skywatchers: A Revised and Updated Version of Skywatchers of Ancient Mexico (University of Texas Press, 2001), p. 144.
[7] H. M. Bricker and V. R. Bricker, in Astronomy in the Maya Codices (American Philosophical Society, Philadelphia, 2011), p. 69.
[8] F. G. Lounsbury, in Maya Numeration, Computation, and Calendrical Astronomy. *Dictionary of Scientific Biography*, vol. 15, suppl. 1, edited by Charles Coulston-Gillispie, (New York: Charles Scriber's Sons, 1978), p. 811
[9] J. S. Justeson, The ancient Maya Ethnoastronomy: An Overview of Hieroglyphic Sources, in World Archaeoastromy: Selected Papers from the Second Oxford International Conference on Archaeoastronomy, edited by A. F. Aveni, (Cambridge University Press, 1989), p.103.
[10] W. A. Saturno, D. Stuart, A. F. Aveni and F. Rossi, Science **336**, 714 (2012).
[11] http://nssdc.gsfc.nasa.gov/Planetary/factsheet/
[12] H. M. Bricker and V. R. Bricker, Current Anthropology **24**, 1-23 (1983).
[13] H. M. Bricker and V. R. Bricker, in Astronomy in the Maya Codices (American Philosophical Society, Philadelphia, 2011), pp. 163, 367, 847.
[14] H. M. Bricker and V. R. Bricker, in Astronomy in the Maya Codices (American Philosophical Society, Philadelphia, 2011), p. 235.
[15] A. F. Aveni, in Skywatchers: A Revised and Updated Version of Skywatchers of Ancient Mexico (University of Texas Press, 2001), pp. 156-157.
[16] E. H. Boone, in Cycles of Time and Meaning in the Mexican Books of Fate (University of Texas Press, 2007), pp. 44-45.
[17] F. Lounsbury, in Dictionary of Scientific Biography, vol. 15, (Scribners, New York, 1978), suppl. 1, p. 787.
[18] T. A. Joyce, in Mexican Archaeology: An Introduction to the Archaeology of the Mexican and Mayan Civilizations of Pre-Spanish America (Cambridge University Press, New York, 1914), p. 64.
[19] L. Schele and D. Freidel, in A forest of Kings (New York William Morrow and Co., 1990), p. 134.
[20] P. J. O'Brien and H. D. Christiansen, American Antiquity **51**, 136 (1986).